14.7 Local Max/Min continued

***Recall: 2nd Deriv. Test*:**

If (a,b) is a critical pt., compute:

$$D = f_{xx}(a,b)f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2$$

1. $D > 0$, $f_{xx} > 0$, $f_{yy} > 0$ → **local min**
2. $D > 0$, $f_{xx} < 0$, $f_{yy} < 0$ → **local max**
3. $D < 0$ → **saddle point**
4. $D = 0$, test inconclusive

Entry Task:

Find and classify all critical points for

$$f(x,y) = x^2y - 9y - xy^2 + y^3$$

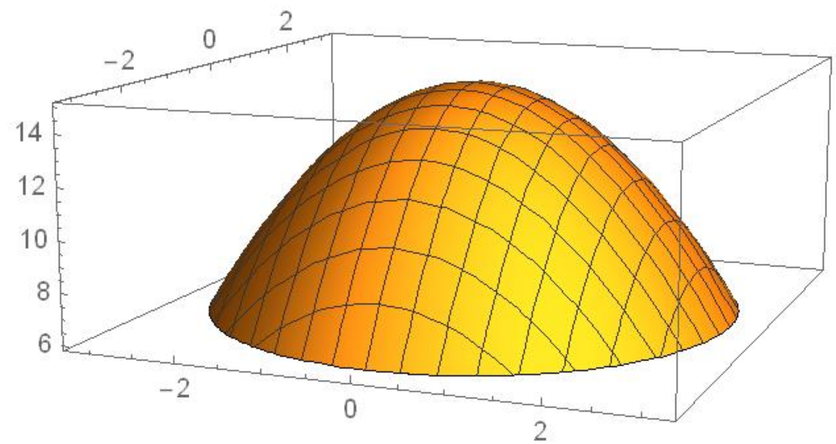
Quick Examples: All three examples have a critical point at (0,0).

1. $f(x,y) = 15 - x^2 - y^2$,

$f_{xx} = -2$, $f_{yy} = -2$, $f_{xy} = 0$

$D = (-2)(-2)-(0)^2 = 4$

$D > 0$, $f_{xx} < 0$, $f_{yy} < 0$

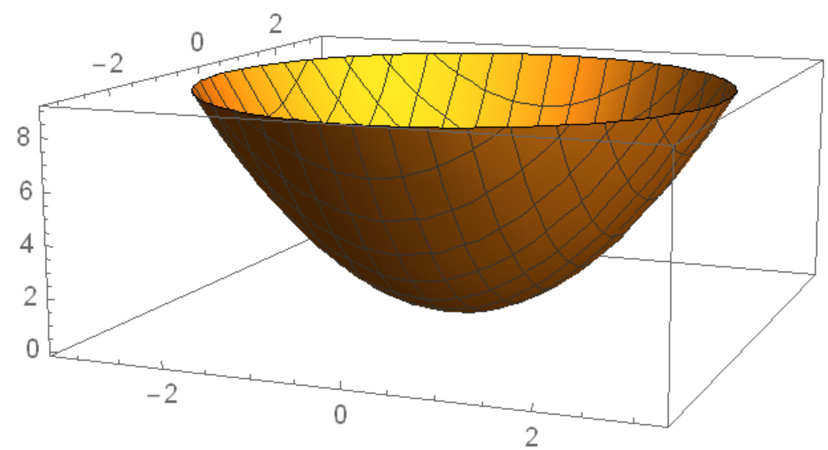


2. $f(x,y) = x^2 + y^2$,

$f_{xx} = 2$, $f_{yy} = 2$, $f_{xy} = 0$,

$D = (2)(2)-(0)^2 = 4$

$D > 0$, $f_{xx} > 0$, $f_{yy} > 0$

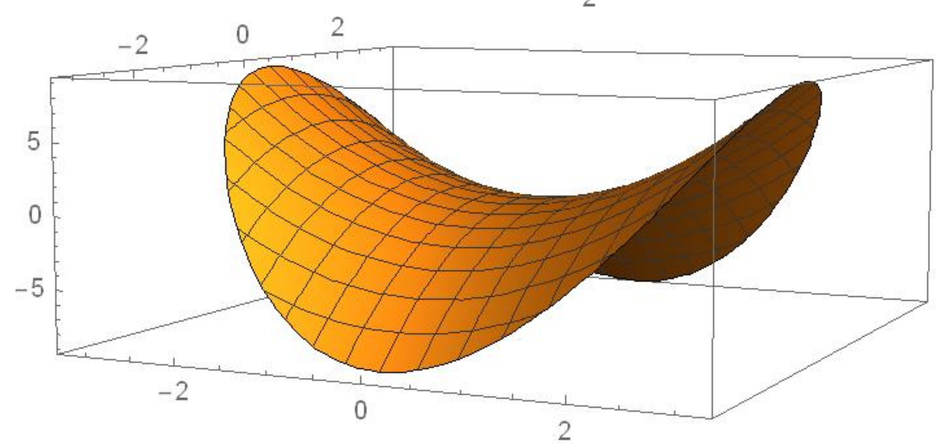


3. $f(x,y) = x^2 - y^2$

$f_{xx} = 2$, $f_{yy} = -2$, $f_{xy} = 0$,

$D = (2)(-2)-(0)^2 = -4$

$D < 0$ (note also, $f_{xx} < 0$, $f_{yy} > 0$)



Global Max/Min: Consider a surface z=f(x,y) over region R on the xy-plane. The **absolute/global max/min** over R are the largest/smallest z-values.

Key fact (Extreme value theorem)
The absolute max/min must occur at

1. A critical point, or
2. A boundary point.

Easy Example: Consider the paraboloid

$$z = x^2 + y^2 + 3$$

above the circular disk $x^2 + y^2 \leq 4$.
Find the absolute max and min.

How to do global max/min problems:

Step 1: Find critical pts inside region.

Step 2: Find critical numbers and corners above each boundary.

Step 3: Evaluate the function at all pts from steps 1 and 2.

Biggest output = global max

Smallest output = global min

Boundaries (step 2) details:

i) For each boundary, give an equation in terms of x and y. Find intersection with surface.

ii) Find critical numbers and endpoints for this one variable function. Label “corners”.

Another Example:

Find the absolute max/min of

$$f(x, y) = x^3 - 12x + y^2$$

over the region

$$x \geq 0, x^2 + y^2 \leq 9.$$

Homework hints

In applied optimization problems,

(a) Label Everything.
(b) ***Objective***: What you are optimizing?!?!
(c) ***Constraint***: What is given?
(d) Use the constraints and labels to give a 2 variable function for the objective.

Then find critical points!!

HW Examples:

1. Find the dimensions of the box with volume 1000 cm^3 that has minimum surface area.

Objective?

Minimize **surface area.**

Constraint?

Given that volume is 1000.

2. Find the points on the cone $z^2 = x^2 + y^2$ that are closest to (4,2,0).

Objective?

Minimize **distance** from (x,y,z) points to the point (4,2,0)

Constraint?

(x,y,z) must be on $z^2 = x^2 + y^2$.

3. You want to build aquariums with slate for the base and glass for the sides (and no top). Slate costs $5 / in^2$ and glass costs $1 / in^2$. If the volume must be 1000 in^3, then what dimensions will minimize cost?

Objective?
Minimize **cost**

Constraint?
Volume needs to be 1000.